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RELATIVISTIC DYNAMICS AND SPACE-TIME STRUCTURE OF FEW-BODY PROCESSES

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1. INTRODUCTION

The time dependent propagator theory appears to be a suitable apparatus for the analysis of space-time features of the relativistic dynamics. We will introduce two kinds of generalizations. The adiabatic hypothesis is substituted by the disturbative adiabatic hypothesis, resulting with the corresponding space-time regionalization of scattering processes. After that we will define a form of the propagator theory for the description of the few-body processes whose propagation is based on the family of spacelike surfaces.

The general scheme can be applied to different models of few body processes also including compositeness levels mixing.

The first part of our investigation includes two broad theoretical approaches.

One of them is the "traditional" approach for studies of scattering processes with bound states of the nucleons involved in a process,¹ based on propagator theory²⁻⁵ with the use of the adiabatic hypothesis,^{4,6,1} generalized Bethe-Salpeter equation⁷⁻⁸ and method of functional derivation.^{5,1} With appropriate modifications and generalizations this kind of approach gives a chance to apply the theory to different classes of scattering processes as well as for various models by which these processes are described.

The other theoretical segment is comparative assessment of approaches for description of few body systems at low and intermediate energies. It includes many body and few body processes presented at the meson baryon level, microscopic models (QCD and QCD - inspired models) and hybrid models as well. Majority of these models are not grounded on the First Principles of the Quantum Field Theory (QFT). In the section 2. we shall make an effort to consider a link between these theoretical developments.

The second part of our investigation is based on the fact that in general forms of the relativistic particle theories, a propagation of a particle system is formulated on the family of spacelike surfaces in Minkowski space. For such "one parameter family of spacelike surfaces filling the whole of space-time, so that one and only one member $\sigma(x)$ of the family passes through any given point x ", introduced by Dyson⁹ we will use the name *Dyson Family of Surfaces* (DFS).

Instead of using DFS as a mere introductory concept of QFT, it will be considered as a representative of the general form of relativistic dynamics.^{10,12} Under such an assumption, the well known forms of relativistic dynamics: instant, light cone and point ones appear to be the particular, rigid realizations of an above mentioned general form.

We notice two ways for approaching the relativistic dynamics based on the DFS. One starts with a time dependent propagator theory for composite systems. This concept should be "translated" from the traditional language of the propagation based on the family of instant surfaces to the presentation of the propagation based on the Dyson family of surfaces.

In this contribution we shall present only main characteristics of our theoretical approach. The complete form including details of applications will be published elsewhere.¹⁰ Some segments of the formalism have been introduced earlier.¹¹⁻¹²

2. EXPANDED TIME-DEPENDENT PROPAGATOR THEORY

The traditional theory which includes the adiabatic hypothesis we shall call the *Standard Time-Dependent Propagator Theory* (STDPT).

Constituents of the process, the particles and their bound states define initial and final channels:

$$\{I_1, \dots, I_k\} = \alpha; \quad \{F_1, \dots, F_l\} = \beta. \quad (1)$$

The amplitudes and propagators are

$$\chi_\alpha(n) \equiv \langle 0 | T(\psi(x_1) \dots \psi(x_n)) | \alpha \rangle; \quad \bar{\chi}_\alpha(n) \equiv \langle \alpha | T(\psi(x_1) \dots \psi(x_n)) | 0 \rangle. \quad (2)$$

$$G_{\alpha\beta}(n; m) = \langle 0 | T \left(T(\psi(x_1) \dots \psi(x_n)) T(\psi(x'_1) \dots \psi(x'_m)) \right) | 0 \rangle. \quad (3)$$

Here, $\psi(x_i)$ are renormalized Heisenberg operators.

For (2) and (3) the limiting procedure of Gell-Man and Low⁸ reads

$$[\psi(x_1) \dots \psi(x_n)]^t = \lim_{x_{io} \rightarrow t} T(\psi(x_1) \dots \psi(x_n)). \quad (4)$$

$$[\psi(x_1) \dots \psi(x_n)]^{in, out} = \lim_{t \rightarrow -\infty, +\infty} [\psi(x_1) \dots \psi(x_n)]^t. \quad (5)$$

Here $\lim_{x_{io} \rightarrow t} T$ is notation for the T product in the limiting instance where two or more times are equated.

One gets the S -matrix element $S_{\beta\alpha} = {}^{out}\langle \beta | \alpha \rangle^{in}$ by introducing the unit operators $\sum_\alpha | \alpha \rangle \langle \alpha |$ and $\sum_\beta | \beta \rangle \langle \beta |$:

$$\begin{aligned} L(\infty, -\infty) G_{\alpha\beta}(n; m) &= \langle 0 | [\psi(x_1) \dots \psi(x_n)]^{out} [\psi(x'_1) \dots \psi(x'_m)]^{in} | 0 \rangle = \\ &= \sum_{\alpha, \beta} \chi_\beta^{out}(n) S_{\beta\alpha} \bar{\chi}_\alpha^{in}(m). \end{aligned} \quad (6)$$

Above definitions allow the traditional formulation of the *Adiabatic Hypothesis*:¹ "No error is introduced into the treatment of the physically realizable scattering process by a formulation of the theory in which the interactions among the particles of interest are 'turned off' at remote times - provided, of course, that the turning-off procedure is sufficiently gradual that it does not, of itself, create disturbances."

The adiabatic hypothesis introduces a particular regionalization of the process in the space-time and also introduces a sort of a cluster decomposition. The propagator (3) can be written in the form

$$G_{\alpha\beta}(n; m) = G_{\beta}(n)R_{\alpha\beta}G_{\alpha} = \prod_{f=1}^l G_f R_{\alpha\beta} \prod_{i=1}^k G_i. \quad (7)$$

The cluster decomposition is seen on the right-hand side of the expression (7), where G_i and G_f are propagators of the constituents of the scattering process and $R_{\alpha\beta}$ is a truncated propagator.

If analogous procedure is applied to the amplitudes by writing

$$\bar{\chi}_{\beta}^{out}(n) = \prod_{f=1}^l \bar{\chi}_{\beta f}; \quad \chi_{\alpha}^{in}(m) = \prod_{i=1}^k \chi_{\alpha i}. \quad (8)$$

and by using (7) and (6), one gets for S -matrix the expression

$$S_{\beta\alpha} = \bar{\chi}_{\beta}^{out} R_{\alpha\beta} \chi_{\alpha}^{in} = \prod_{f=1}^l \bar{\chi}_{\beta f} R_{\alpha\beta} \prod_{i=1}^k \chi_{\alpha i}. \quad (9)$$

Space-time content of the adiabatic hypothesis is a segmentation of the scattering process to the initial, interactional and final phases. Consequently, in the STDPT, propagator (7) and S -matrix (9) are factorized on the corresponding propagation regions $\mathcal{R}_i, \mathcal{R}_k$ and \mathcal{R}_f respectively. These regions are separated by two space-like surfaces.

Our view on the scattering processes will be somewhat different. We shall retain the supposition that interaction between constituents of scattering processes could be switched off at the remote times, or precisely for the incoming and outgoing space-time regions of propagation. But in addition we shall include a new one by which the mechanism of inclusion/exclusion is such that it creates the disturbances for incoming/outgoing channels. We shall suppose also that such effects can be represented by segments obtained in a factorization of the scattering picture i.e. that the transition from incoming/outgoing regions of propagation to the region of "pure" interaction, generally, is not direct but it goes over the intermediate disturbed phases.

Therefore we formulate the *Disturbative Adiabatic Hypothesis* (DAH): In the framework of the QFT, propagation process can be, generally, represented by a few-step mechanism. Finite intermediate regions of disturbance which can be represented by a superposition of the physically realizable configurations for the corresponding channels are compatibly connected with the central, interaction region. Interaction among the constituents of the scattering process is "turned-off" for the incoming and outgoing propagation regions, so that initial and final configurations are corresponding to them.

The motivations for the above reformulation are mainly the following.

Firstly, they are of the conceptual nature. The intermediate phases have been tacitly already introduced (by using, for example, off mass shell contributions) and

different cluster decompositions are often introduced ad hoc without explication of whether it is consistent or not with the general procedure.

Secondly, in applications of the standard formulation one often finds the incompatibility of the interaction effects originating from scattering amplitudes and from truncated propagators. That can be easily seen in the analysis of the $\gamma D \rightarrow pn$ process.¹³

Thirdly, the idea was to find consistent and in the same time flexible formulation which could be applied to many concrete scattering situations where these ones appear as particular cases.

In the *Expanded Time-Dependent Propagator Theory* space-time content will be correspondingly richer. Two additional domains of scattering process appears. Disturbed channels of the initial and final configurations correspond to the incoming and outgoing disturbance regions \mathcal{R}_{id} i.e. \mathcal{R}_{fd} . So one can visualize the sequence of the five regions: $\mathcal{R}_i, \mathcal{R}_{id}, \mathcal{R}_k, \mathcal{R}_{fd}$ and \mathcal{R}_f separated by the four space-like surfaces. DAH causes the corresponding factorization of the representatives of the scattering theory.

In our formulation the propagator for the process $\alpha \rightarrow \beta$ is written

$$G_{\alpha\beta} = G_\beta \bar{K}_\beta^d R_{\alpha\beta} K_\alpha^d G_\alpha. \quad (10)$$

Here \bar{K}_β^d and K_α^d correspond to the effect of disturbance introduced. With them the generalized expanded interaction term $M_{\alpha\beta}$ reads

$$M_{\alpha\beta} = \bar{K}_\beta^d R_{\alpha\beta} K_\alpha^d. \quad (11)$$

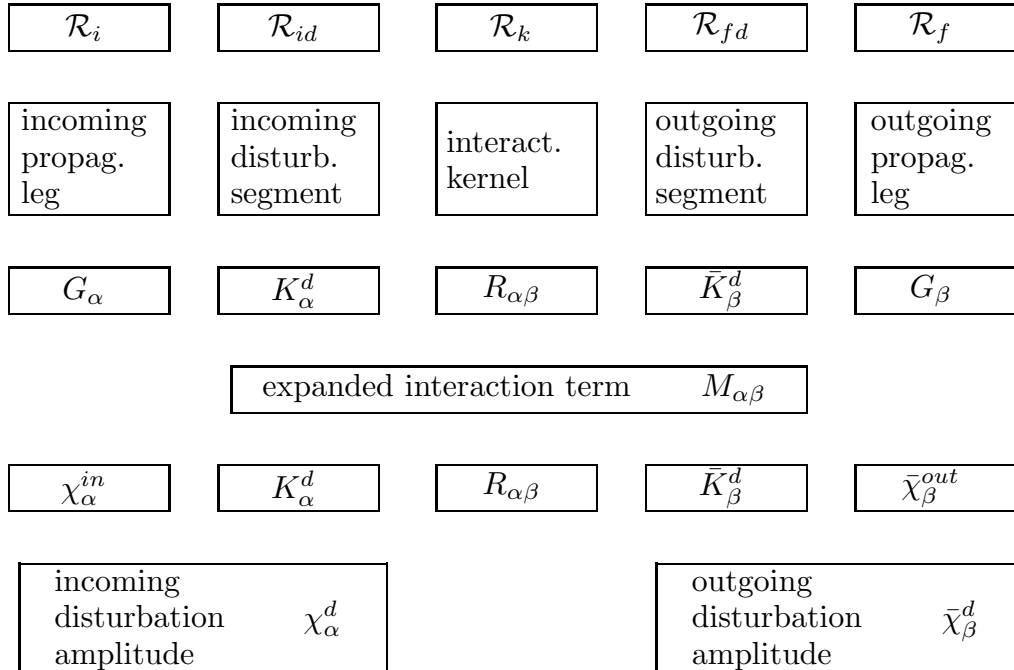
The S -matrix elements gets the form

$$S_{\beta\alpha} = \bar{\chi}_\beta^{out} \bar{K}_\beta^d R_{\alpha\beta} K_\alpha^d \chi_\alpha^{in}, \quad (12)$$

and the new objects, "disturbance amplitudes" read

$$\chi_\alpha^d \equiv K_\alpha^d \chi_\alpha^{in}; \quad \bar{\chi}_\beta^d \equiv \bar{\chi}_\beta^{out} \bar{K}_\beta^d. \quad (13)$$

The process defined by DAH is visualized by the following scheme



First line corresponds to the domains of the process, second to the propagational and interactional content of the segments, third to the propagator and fifth to the S -matrix. Forth and sixth line illustrate the joining of disturbance effects.

The contact with experimental situation is model dependent. The interaction is given with the form of the expanded interaction term $M_{\alpha\beta}$. In addition it is necessary to define the disturbance effects. The remote time amplitudes χ_α^{in} and $\bar{\chi}_\beta^{out}$ are determined phenomenologically.

The general formalism contains also two procedures which are independent and mutually compatible.

One of them consist in trying to find "natural" representation of disturbance effects in the form

$$K^d = G^d V; \quad \bar{K}^d = \bar{V} G^d. \quad (14)$$

Propagators G^d are related to the finite domains. They have the same physical meaning as propagational segments of a truncated propagator $R_{\alpha\beta}$ which appear in the course of the functional derivation.

The second procedure consists in the series expansion of $M_{\alpha\beta}$ and reads

$$M_{\alpha\beta} = (\bar{K}_\beta^{d(0)} + \bar{K}_\beta^{d(1)} + \dots)(R_{\alpha\beta}^{(0)} + R_{\alpha\beta}^{(1)} + \dots)(K_\alpha^{d(0)} + K_\alpha^{d(1)} + \dots). \quad (15)$$

This expansion is not perturbative series but it reflects the content of Bethe-Salpeter equation with the precise meaning of spatio-temporal composition of interaction effects.

One needs an additional analysis of terms in order to achieve a compatibility of the interaction contributions independently of their origin.

3. PROPAGATION ON THE DYSON FAMILY OF SURFACES

As a second aspect of a time dependent presentation of the scattering processes we shall consider (I) the covariant generalization of the space-time propagation of the constituents of a scattering process, and (II) the analysis of the causality conditions which appear in connection with the mentioned generalization.

(I) One uses a one parameter DFS $f^a(\sigma)$. For the members of the family of manifolds in Minkowski space the inclusion relation $\sigma < \sigma_o$ is defined. This allows us to generalize the time step function by defining chronological step function Θ_σ which appears as a distribution in Minkowski space. Analogously, generalized chronological product T_σ is introduced.

By the above constructs one defines in a general manner the propagator G and the amplitudes χ and $\bar{\chi}$ for a particular channel, i.e.

$$G_{\alpha\beta}(n; m) = \langle 0 | T_\sigma \left(T_\sigma(\psi(x_1) \dots \psi(x_n)) T_\sigma(\psi(x'_1) \dots \psi(x'_m)) \right) | 0 \rangle, \quad (16)$$

$$\chi_\alpha(n) = \langle 0 | T_\sigma(\psi(x_1) \dots \psi(x_n)) | \alpha \rangle; \quad \bar{\chi}_\alpha(n) = \langle \alpha | T_\sigma(\psi(x_1) \dots \psi(x_n)) | 0 \rangle. \quad (17)$$

which are covariant generalizations of the usual ones.

The above presentation is realized in such a manner that allows us to perform time dependent factorization of the scattering processes.

(II) The implicit correspondence between relevant families of the space-like surfaces and specific scattering processes is a distant reflection of the connection between geometry and dynamics which is practically impossible to study by a standard techniques of the QFT.

Additional results could be achieved by considering the causality conditions. For the propagator of scattering processes only one condition is imposed by temporal T or chronological T_σ product. Thregulates the succession of the events. But a propagator contains non causal

contributions as well. It includes time-like separation of the constituents going to interact and arbitrarily large space-like separation of the basic constituents of the bound states. This poses the well known problem of scattering theory of composed systems.

If we take into account nonlocality of constituents we can consider causal propagation by introducing a specific cut-off.

Mathematically, one can introduce the class of the families of space-like surfaces $\Sigma^A(\sigma) = \cup_{a \in A} f^a(\sigma)$ for which one defines the characteristic function P_{Σ^A} . We shall impose that the parameterization of families which form a class is continuous and finite. If one defines the corresponding space-like domains $D^A = (\sigma \in f^a(\sigma) \mid a \in A)$ then to the family of the continuously deformable domains $\Sigma(D^A)$ corresponds the class of the families of space-like surfaces $\Sigma^A(\sigma)$.

The propagators defined through the corresponding characteristic functions remains functionals whose arguments lie in the whole space M^n . However, the significant contributions are only those coming from the causal separation of their arguments.

The fact which we would like to describe quantitatively is the following: in the course of the scattering process its constituents, the particles and the bound states, are in the corresponding space-time domains. So, it is natural to demand that propagators and amplitudes reflect mutual correlations and collective effects coming from the interaction.

In addition, we expect compatibility between the disturbance mechanism and the outlined one. As a consequence all the recoil and retardation effects which have unpleasant feature in the standard formulation appear now in the common footing with the whole dynamics.

4. APPLICATIONS

An example is the coherent pion photoproduction on deuteron: $\gamma d \rightarrow \pi d$. By using (12), (11) and development (15) and by keeping the zeroth and first order of the meson exchange terms, the S -matrix (12) gets the form:

$$S_{\beta\alpha} = \bar{\chi}_\beta^{out} (\bar{K}_\beta^{d(0)} R_{\alpha\beta}^{(0)} K_\alpha^{d(0)} + \bar{K}_\beta^{d(0)} R_{\alpha\beta}^{(1)} K_\alpha^{d(0)} + \bar{K}_\beta^{d(0)} R_{\alpha\beta}^{(0)} K_\alpha^{d(1)} + \bar{K}_\beta^{d(1)} R_{\alpha\beta}^{(0)} K_\alpha^{d(0)}) \chi_\alpha^{in}. \quad (18)$$

As a result of the application of the DAH all first order terms are mutually compatible and consequently it is possible to make their consistent resummation. We note that it is not the case in the standard procedure (not using DAH), for example for $\gamma d \rightarrow pn$ process.¹³

The main result is the new structure of the transition operator expressed through the one-nucleonic variables.

For a given energy region one has to specify the disturbance input, in particular the interaction kernel and the type of meson-nucleon coupling.

If one considers the energy region of the first few pion nucleon resonances one might choose that exchanged meson is a pion and the coupling is a pseudoscalar one. Inclusion of these assumptions leads to the model studied.^{14,15} The structure of S -matrix appears to be a superposition of the impulse approximation, rescattering

of pion on nucleons, and double pion photoproduction on nucleon with subsequent absorption of one pion by other nucleon.

The above exposed theoretical scheme is so general that it allows its application to reactions at QCD level as well as for hybrid models. Details will be published elsewhere.

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